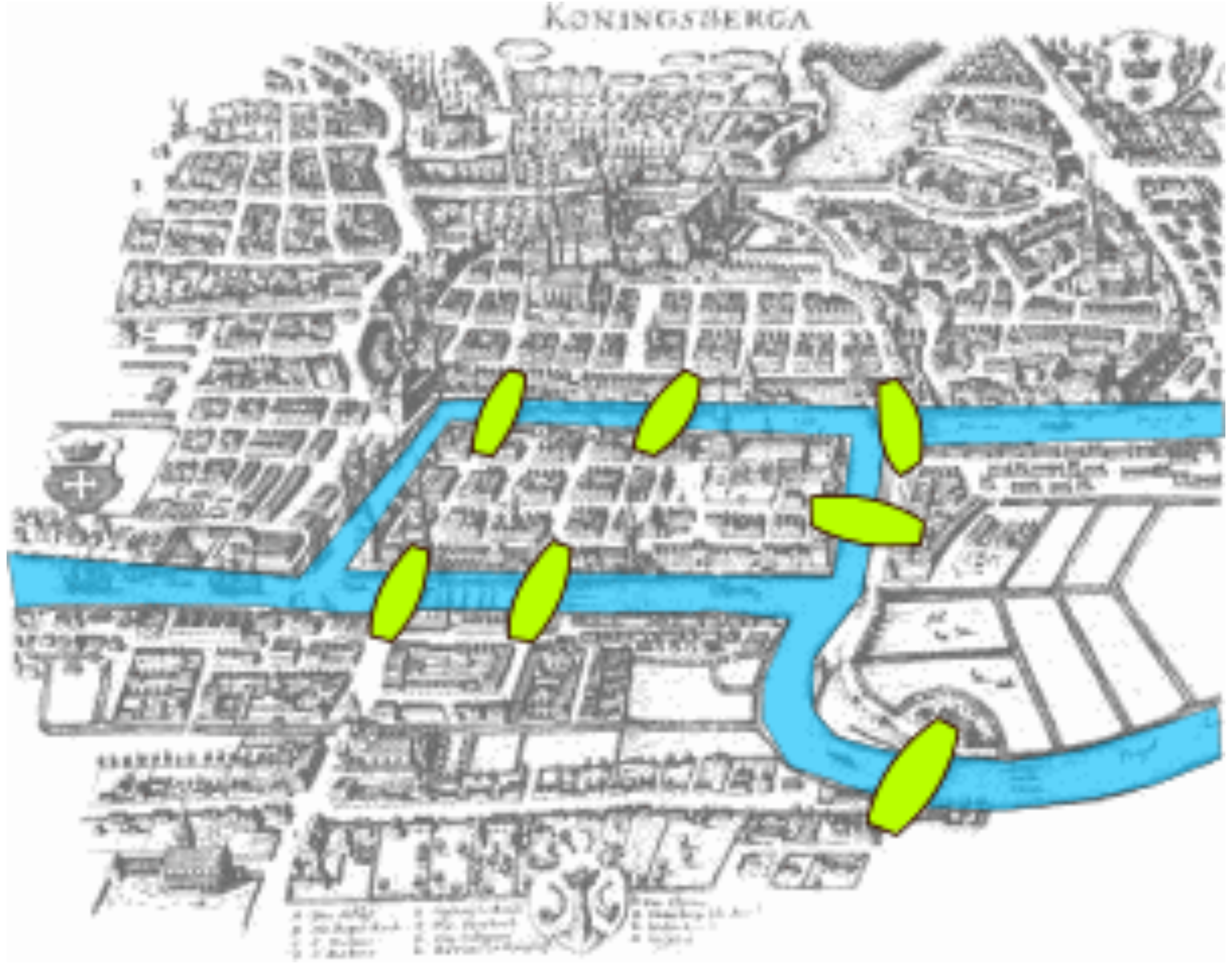
Agent-Based Simulation in Complex Networks

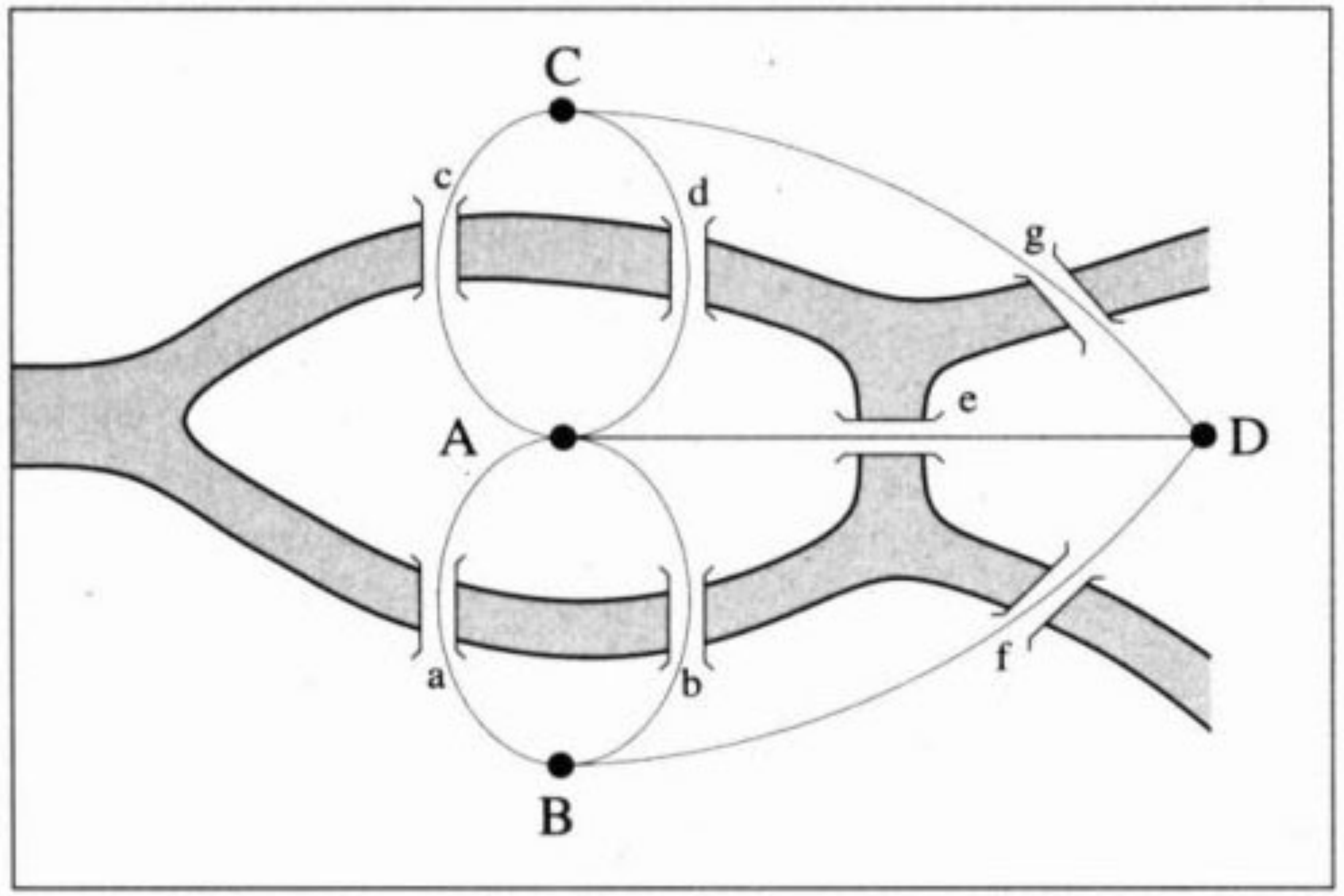
ESSAI 2024. Athens

Miguel Rebollo (@mrebollo)

Session 2. Complex networks

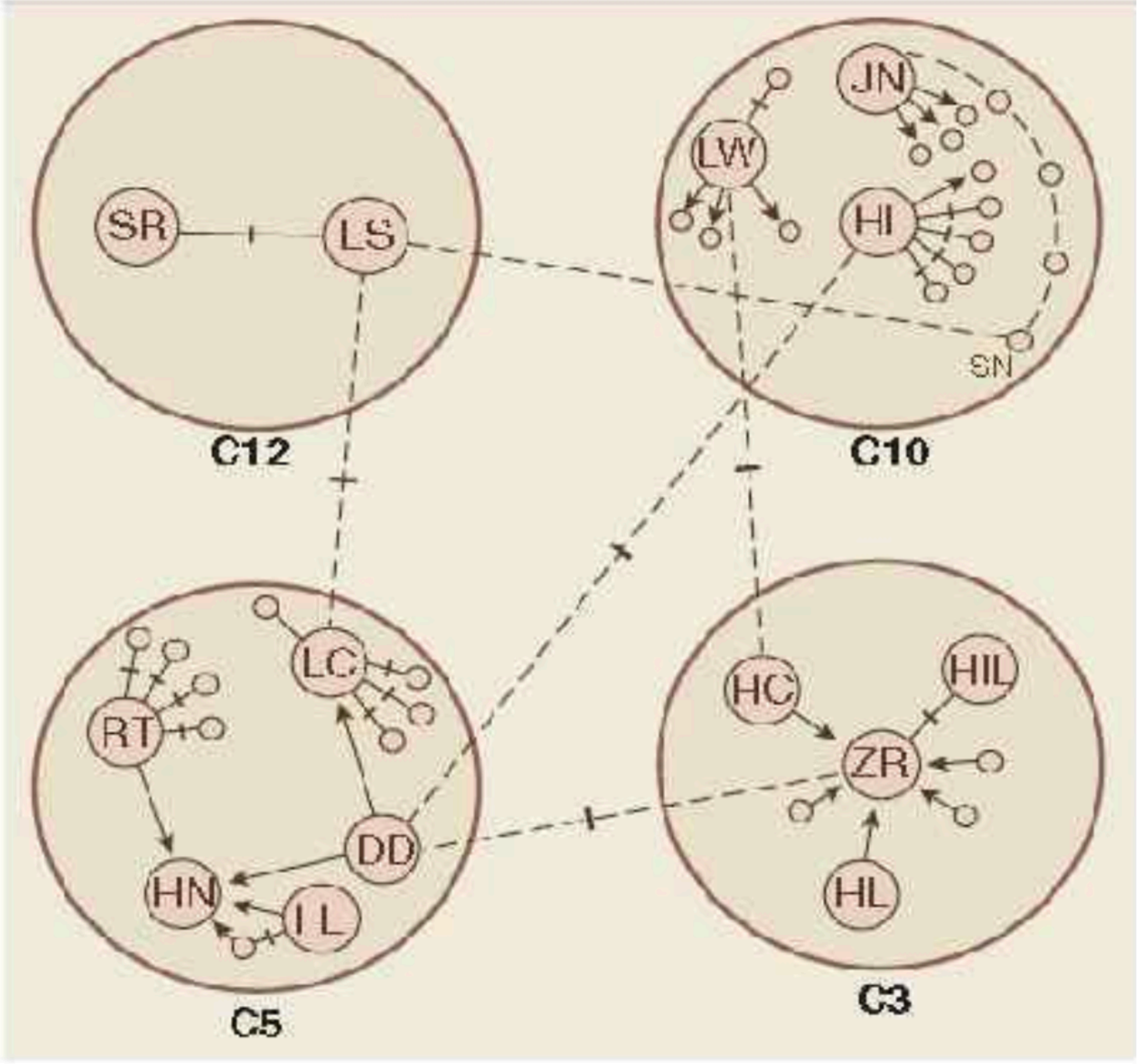
Origin





The Konigsberg bridge problem (Euler, 1736). The problem was to find a walk through the city that would cross each bridge once and only once.

First application to human beings

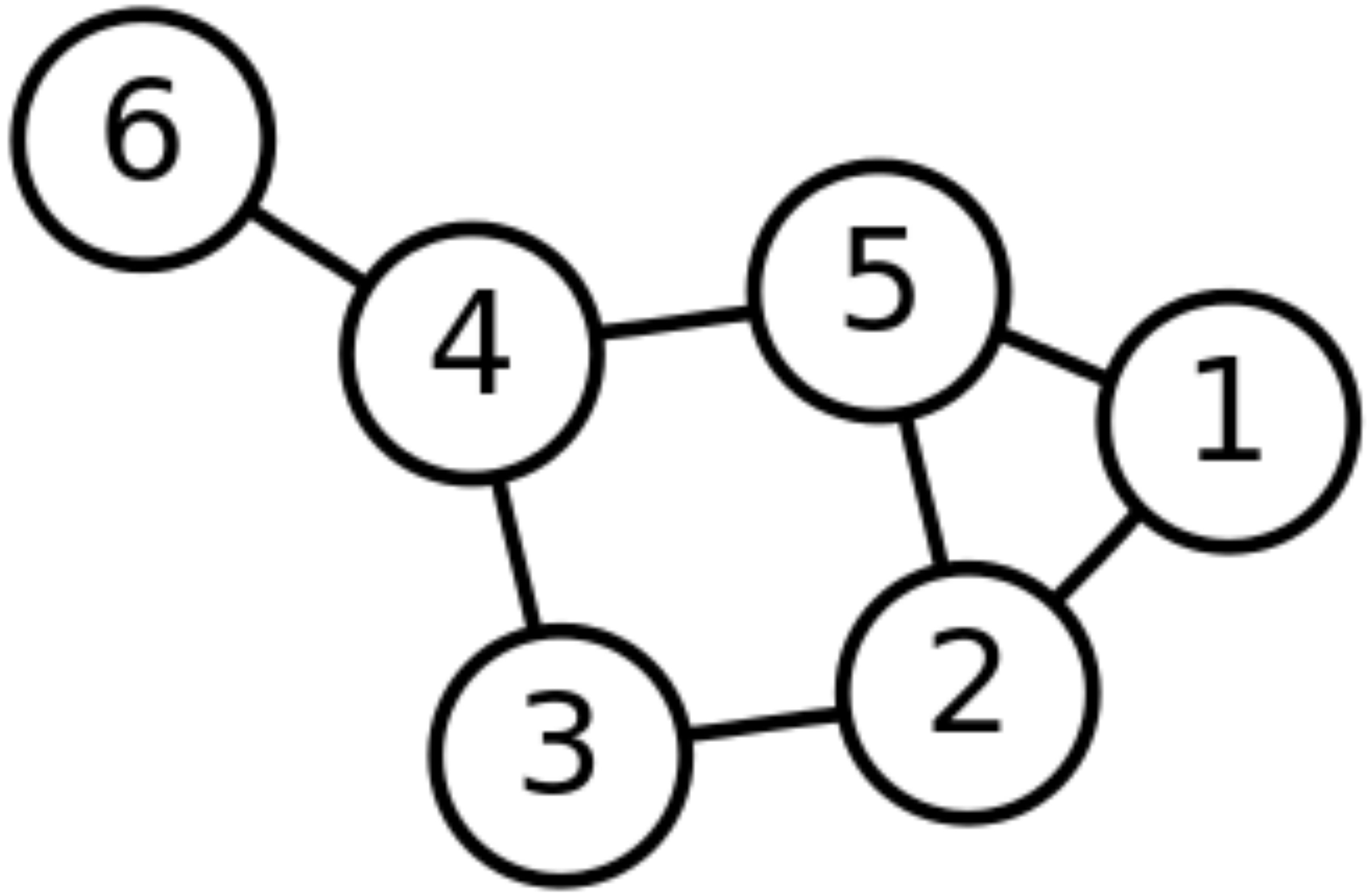


Used to study why a group of girls that run away from a college at Hudson (NY St) in 1934 (Jacob Moreno).

The relations among them and the cottages they live in where represented

But we’ve discovered that networks are different

Graph representation

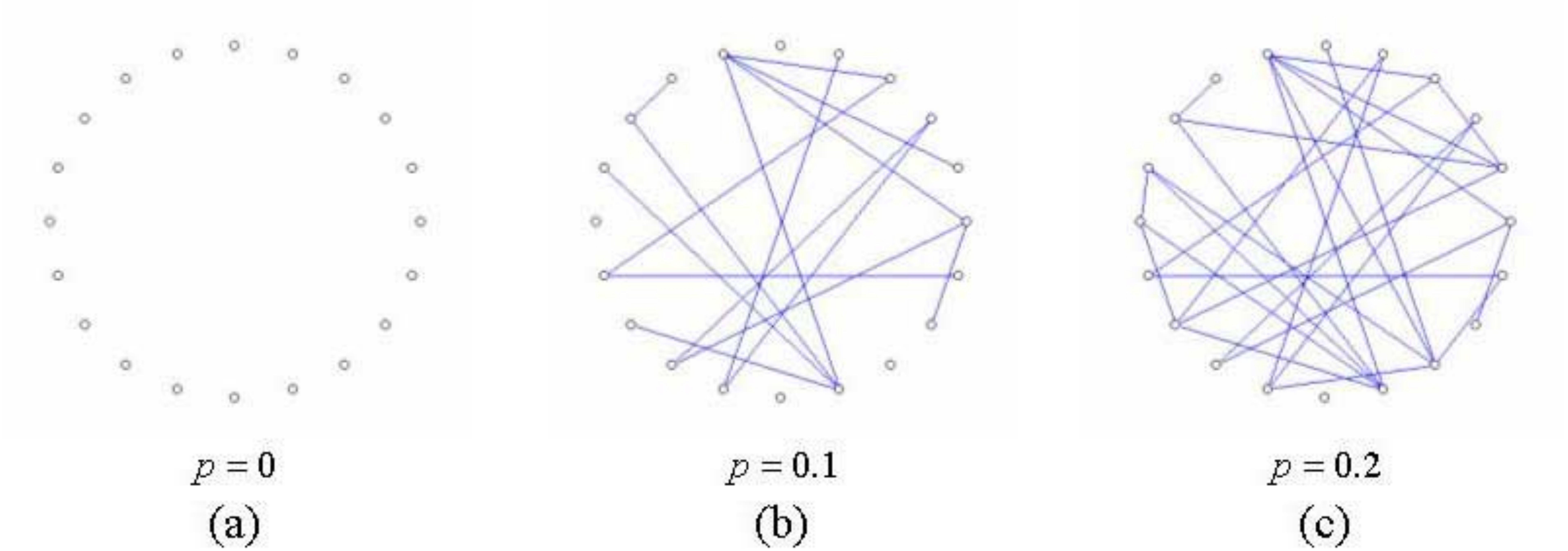


$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Basic concepts

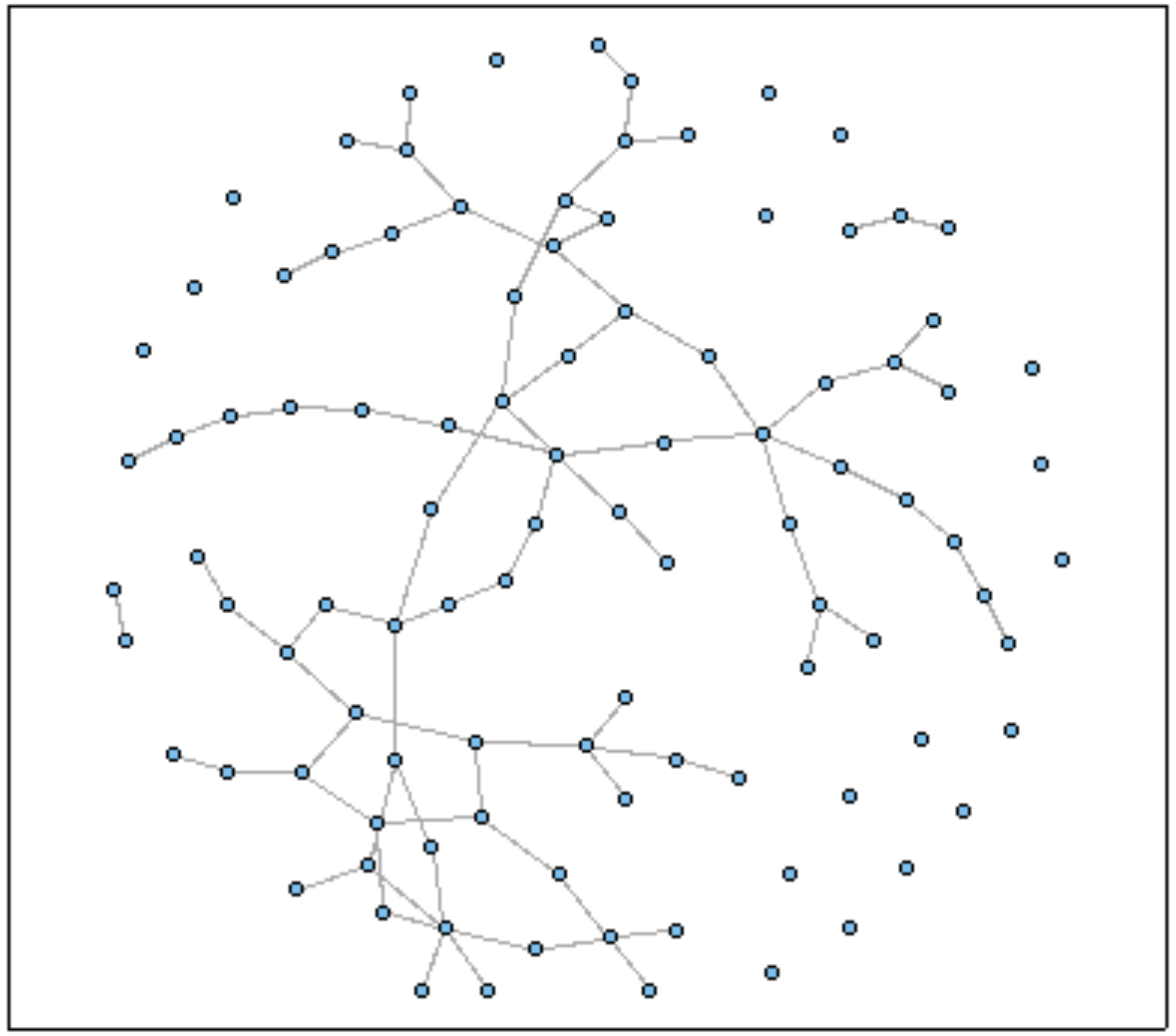
- **Shortest path** d_{ij} minimal distance (weight) between nodes i and j
- **Average path length** l average of d_{ij} between all the nodes
- **Diameter** D the longest (maximum) of the shortest paths max d_{ij}
- **Degree** d_i number of neighbors of the node i
- **Clustering coefficient** C number of triangles of all the possible ones

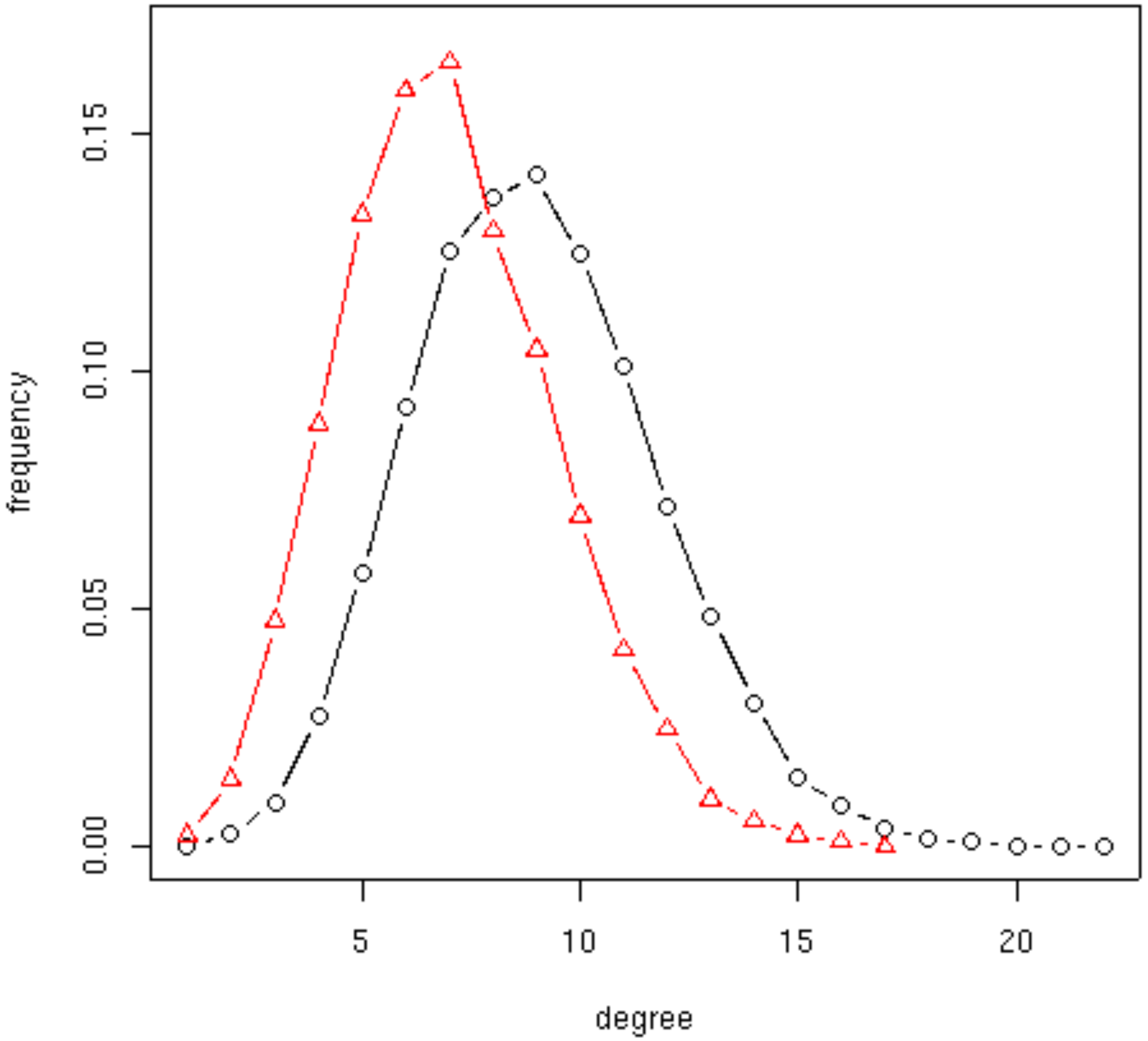
Random graphs. Erdös-Renyi model (1959)



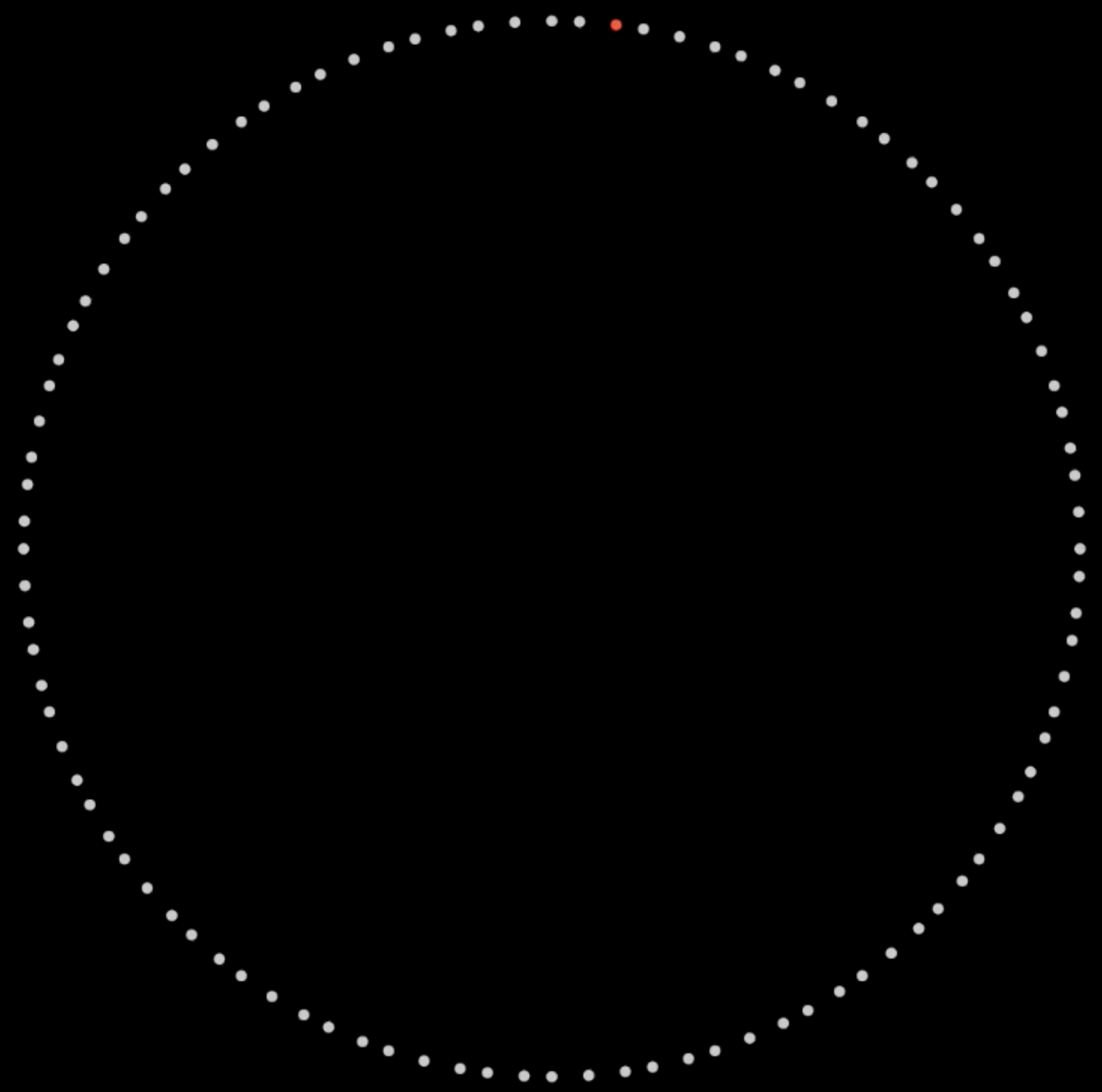
- A new edge is added between two nodes with probability p.
- Emergence of the **giant component**: when p > 1 / n
- The complete network is connected after n log n edges

Random graphs. Degree distribution





The degree distribution of the Erdös-Renyi model is a Poisson one



Random networks. Giant component

6 degrees of separation

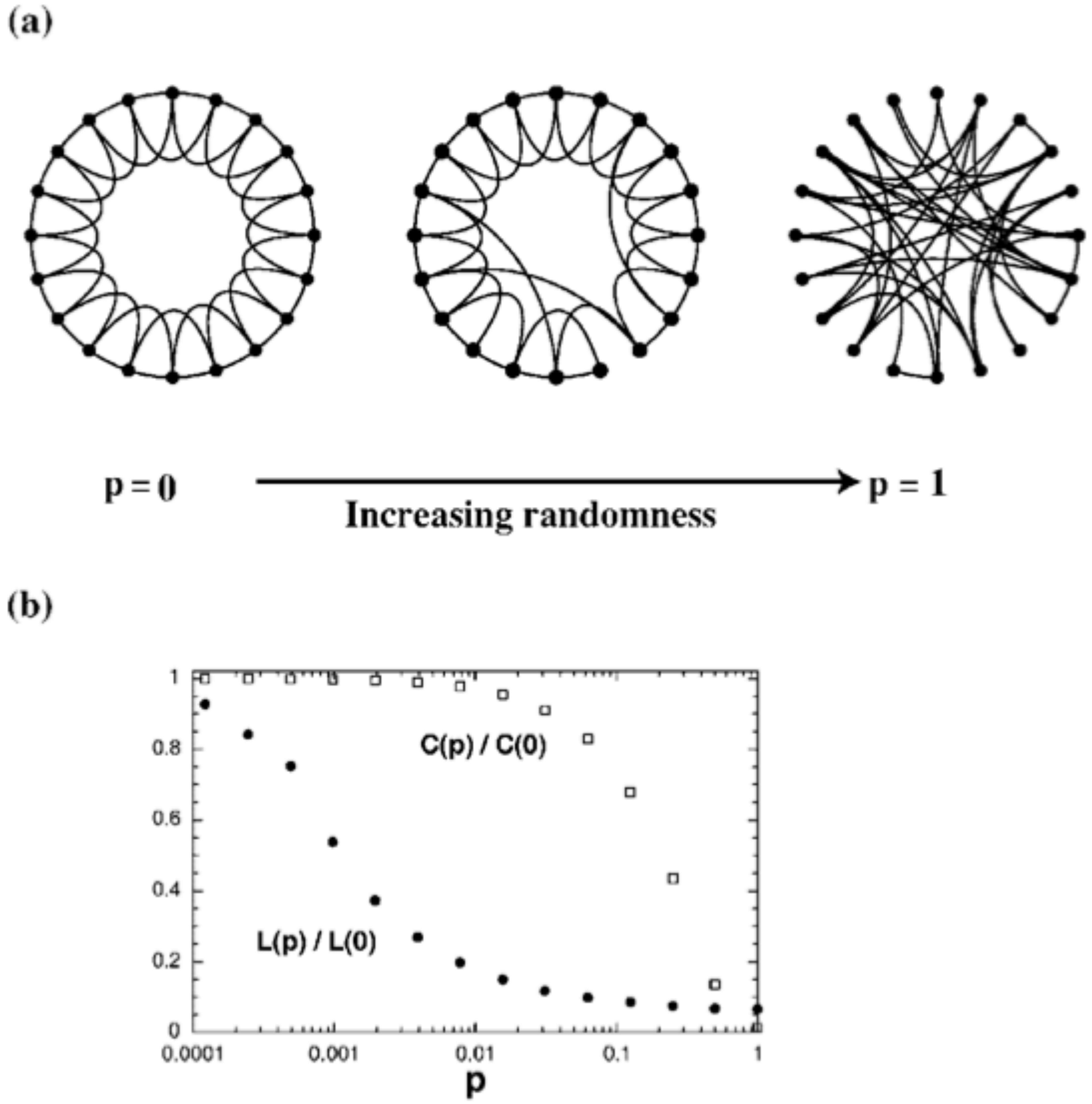
First reference: Short Story ’Chains’ (F. Karinthy, 1929)

Milgram’s experiment (1967).

- people at Omaha, Nebraska and Wichita was chosen
- they have to send a letter to one person in Boston or Massachusetts
- if they know the target, they send them the letter
- if not, they send it to an friend who is more likely to know the target
- 64 letters reached the target using between 2 an 10 steps → average path length in [5.5, 6]

Samples: Kevin Bacon or Erdös numbers

Small world network



The Watts-Strogatz model.

It begins with a regular lattice and rewires edges at random.
Small-world effect appears due to

- high clustering
- short path lengths

...but it still has a Poisson degree distribution